**9.2 Combining Two Functions: Sums and Differences**

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| **A Definitions**<br><br>The sum and difference of two functions are defined by<br><br>$$(f+g)(x) = f(x) + g(x)$$<br>$$(f-g)(x) = f(x) - g(x)$$ | Ex 1. Let $f(x) = x^2 - 1$ and $g(x) = 1 + \sqrt{x+1}$ . Find<br><br>a)  $(f+g)(0)$<br><br><br>b)  $(f-g)(3)$<br><br><br>c)  $(f+g)(-2)$<br><br><br>d)  $(f+g)(x)$ |
| **B Domain of Sum and Difference of Two Functions**<br><br>The domain of the sum or difference of two functions is the intersection of their domains.<br><br>$$D_{f \pm g} = D_f \cap D_g$$ | Ex 2. For each case, find the domain of the sum of the given functions.<br><br>a)  $f(x) = x$  ;  $g(x) = 1/x^2$<br><br><br>b)  $f(x) = \sqrt{2-x}$  ;  $g(x) = \log(x+1)$<br><br><br>c)  $f(x) = 2^{x-2}$  ;  $g(x) = x^3 + 1$<br><br><br>d)  $f(x) = \sqrt{4-x^2}$  ;  $g(x) = \dfrac{1}{x-1}$  ;  $h(x) = \log x$ |

## C Point by Point

Evaluate $f \pm g$ at every possible number $x$.

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Ex 3. Given

$f = \{(1,0), (0,-1), (-1,2)\}$
$g = \{(0,1), (2,-1), (1,3)\}$

Find $f + g$.

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Ex 4. The functions $f$ and $g$ are given by their graphs on the right figure. Graph the function $f - g$.



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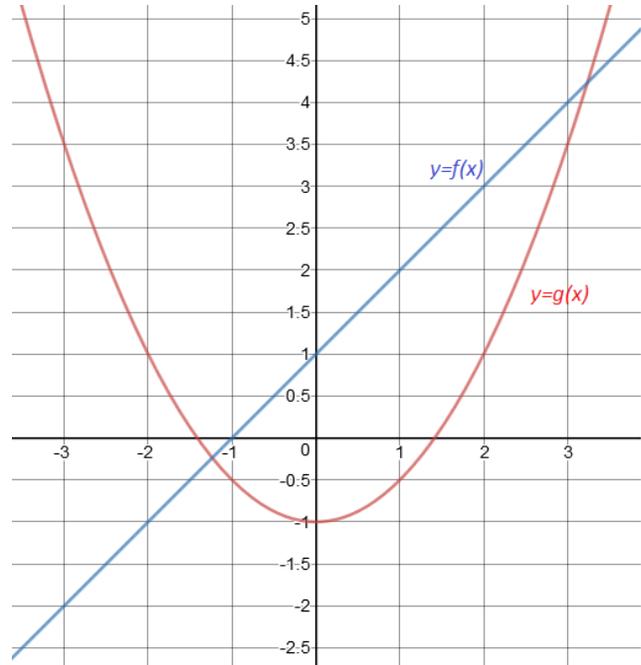
Ex 5. Complete the following table. Justify your reasoning.

|  | $f + g$ | even | odd |
|---|---|---|---|
|  | even |  |  |
| $g$ | odd |  |  |

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Ex 6. For each case, justify your answer.

a) Is the sum of two polynomial functions a polynomial function?

b) Is the difference of two rational functions a rational function?

c) Is the sum of two sine functions a sine function?

d) Is the sum of two periodic functions a periodic function?

| Ex 7. Write the following functions as a sum or a difference of two other functions.<br><br>a) $f(x) = \dfrac{1}{x^2 - 9}$ | b) $f(x) = \log \dfrac{x}{x+1}$<br><br><br><br><br>c) $f(x) = \sin(x - \pi/4)$ |
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| Ex 8. Is the sum of two increasing functions, increasing, decreasing or neither? Give examples to justify your answer. | Ex 9. If $Z_f = \{1,2,3\}$ is the set of all zeros of the function $f$ and $Z_g = \{0,1,2\}$ is the set of all zeros of the function $g$, what could you say about the set of all zeros of the function $f + g$? Explain your reasoning. |
| Ex 10. Let $f(x) = \sin x$ and $g(x) = \cos x$. Write $f + g$ and $f - g$ as:<br><br>a) a single sine function<br><br><br><br>b) a single cosine function | Ex 11. The rational function $y = f(x)$ has a horizontal asymptote $y = 5$ and the rational function $y = g(x)$ has a horizontal asymptote $y = -3$. What could you say about the horizontal asymptote of the functions:<br><br>a) $f + g$<br><br><br><br>b) $f - g$ |

**Reading**: Nelson Textbook, Pages 521-528
**Homework**: Nelson Textbook, Page 528 #3, 4, 5, 7, 10, 13, 16